



Answer following Questions:

Question (1) (14 points)

(a) The Bessel equation of order zero is $x^2 y'' + xy' + x^2 y = 0$ show that the roots of indicial

equation are $s_1 = s_2 = 0$ and one solution for $x > 0$ is $J_0 = \sum_{n=0}^{\infty} \frac{(-1)^n}{(n!)^2} \left(\frac{x}{2}\right)^{2n}$.

(b) Show that $\int_0^{\pi/2} J_0(x \cos \theta) \cos \theta d\theta = \frac{\sin x}{x}$

Question (2) (14 points)

(a) Prove that $\int_0^{\pi/2} \cos^{2m-1} \theta \sin^{2n-1} \theta d\theta = \frac{\Gamma(m)\Gamma(n)}{2\Gamma(m+n)}$ where $\Gamma(n)$ is the Gamma function

(b) Evaluate $\int_0^2 x \sqrt[3]{8-x^3} dx$

Question (3) (14 points)

(a) Evaluate $\int_{-1}^1 x P_m(x) P_{m-1}(x) dx$ where P_m is Legendre polynomial of degree m .

(b) Show that the function $u = e^x \sin y$ is harmonic function and find the function v such that $f = u + iv$ satisfy Cauchy-Riemann equations.

Question (4) (24 points)

(a) Solve the differential equation $y'' + 2y' + 2y = 0$ given that $y(0) = 0, y'(0) = 1$

(b) Find Laplace transform of the periodic function $f(t) = \begin{cases} 1 & 0 < t < a \\ 0 & a < t < 2a \end{cases}$ period $2a$

(c) Find inverse Laplace for $F(s) = \frac{1}{(s+1)^4} + \frac{s-3}{s^2-6s+15}$

Question (5) (14 points)

(a) Evaluate $\int_{(3,0)}^{(-3,0)} \frac{(2z-3)}{z} dz$ on the circle $|z| = 3$.

(b) Find the complex integration $\int_C \frac{(z^2+3z)}{(z-2)} dz$, on the circle $|z| = 5$.